Primary, Concrete, Abstract, Formal, Systematic, and Metasystematic Balance Beam Problems. 1995 Pilot Versions

Michael L. Commons                                      Eric Goodheart                                      Francis A. Richards

For Educational Purposes                                3c-6

Consent Form

I, the undersigned, understand that the basic intent of these tasks is to understand reasoning and how it develops. This is entirely voluntary and I may stop at any time. I understand that the information I have given regarding my personal history, as well as the responses I give to the tasks, will be kept strictly confidential. I understand that my identity and personal history will not be divulged in any discussion, student report, lecture, address, or publication derived from this project.

Signature                                          Month      Day      Year

Instructions

Please answer all parts of this questionnaire. Read each section and answer the questions in the order given. Do not go to the following section before you have finished the previous section. Remember this is not a test of your ability as an individual. Rather, we wish to know how adults, in general, reason about the issues presented here. The order of answering is essential to this study about adult reasoning.

What is most important is not only what you consider to be good, but the reasons for your choices. Therefore, it is essential that you state your reasons why you make the choices that you make.

47:14;05-08-95-20 (Saturday, August 5, 1995; 2:47pm)
46:09;02-08-95-20 (Wednesday, August 2, 1995; 9:46am)
58:23;27-07-95-20 (Thursday, July 27, 1995; 11:58pm)
53:12;27-07-95-20 (Thursday, July 27, 1995; 12:53pm)
14:28;24-05-95-20 (Wednesday, May 24, 1995; 2:28pm)
29:18;20-02-95-20 (Monday, February 20, 1995; 6:29pm)
19:15;13-07-94-20 (Wednesday, July 13, 1994; 3:20pm)
21:35;02-20-86-20 (Thursday, February 20, 1986)

© June, 1978, 1979, 1983, 1985, 1994, 1995 Dare Association, Inc., Cambridge, MA. All rights reserved in all parts and accessories. No part of the Manual or of the test, answer sheets, protocols, and other scoring forms, norms, scales, scoring keys, and other accessories associated with it may be printed or reproduced by any other means, electronic, mechanical, or photographic, or portrayed, translated, or included in any information storage and retrieval system, or used to print or otherwise reproduce a computer generated interpretation, without permission in writing from the publisher, Dare Association, Inc. 234 Huron Avenue, Cambridge, MA 02138-1328.
# Primary, Concrete, Abstract, Formal, Systematic, and Metasystematic Balance Beam Problems

<table>
<thead>
<tr>
<th>Date of Birth:</th>
<th>Month</th>
<th>Day</th>
<th>Year</th>
<th>Subject Number</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Place of Birth:</td>
<td></td>
<td></td>
<td></td>
<td>Last four digits of Social Security Number</td>
<td></td>
</tr>
<tr>
<td>City</td>
<td>Country or State</td>
<td>Place of Residence:</td>
<td>City</td>
<td>Country or State</td>
<td></td>
</tr>
<tr>
<td>Sex:</td>
<td>M</td>
<td>F</td>
<td>Zip code:</td>
<td>Age:</td>
<td></td>
</tr>
<tr>
<td>If married:</td>
<td>1st</td>
<td>2nd</td>
<td>3rd</td>
<td>4th</td>
<td>marriage</td>
</tr>
<tr>
<td>If not married:</td>
<td>Single</td>
<td>Divorced</td>
<td>Spouse deceased</td>
<td>Domestic partner</td>
<td></td>
</tr>
<tr>
<td>Child in family:</td>
<td>1st</td>
<td>2nd</td>
<td>3rd</td>
<td>4th</td>
<td>Other</td>
</tr>
<tr>
<td># Sisters</td>
<td>Father's occupation:</td>
<td>Mother's occupation:</td>
<td>Spouse's occupation:</td>
<td>Your occupation:</td>
<td></td>
</tr>
<tr>
<td># Brothers</td>
<td>Father's education:</td>
<td>Mother's education:</td>
<td>Spouse's education:</td>
<td>Your previous occupation:</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Your</th>
<th>Years in school</th>
<th>Major/Minor</th>
<th>Year graduated</th>
<th>Degree earned</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade school</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>High School</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>College</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Graduate School</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Month/Day/Year</th>
<th>Experimenter Name</th>
<th>Experimenter Telephone</th>
<th></th>
</tr>
</thead>
</table>

Methods of administration:
- Mailed to subject who wrote answers
- Handed out to subject who wrote answers
- Subject wrote answers while experimenter was present
- Subject wrote answers and responded to experimenter probes
- Subject wrote answers while experimenter probed and taped
- Experimenter wrote answers
- Experimenter wrote answers and probed
- Experimenter taped and probed for answers
Task P1

This is a balance beam.

M The numbers in the circles below the beam are the weights.
M The numbers to the left and right of the balance point (fulcrum, ∆) are the distances:
M The value x is the amount of weight needed to balance the beam. In the example, the amount needed is 4.

Example

<table>
<thead>
<tr>
<th>3</th>
<th>2</th>
<th>1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>x</td>
<td></td>
</tr>
</tbody>
</table>

M What number makes the beam balance?

Solution

<table>
<thead>
<tr>
<th>3</th>
<th>2</th>
<th>1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>∆</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

M What weight will make the beam balance? That weight goes at the question mark.

Circle the correct answer below:

1. 

<table>
<thead>
<tr>
<th>3</th>
<th>2</th>
<th>1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>x</td>
<td></td>
</tr>
</tbody>
</table>

   a. 0  e. 4
   b. 1  f. 5
   c. 2  g. 6
   d. 3  h. 7

2. 

<table>
<thead>
<tr>
<th>3</th>
<th>2</th>
<th>1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>x</td>
<td></td>
</tr>
</tbody>
</table>

   a. 0  e. 4
   b. 1  f. 5
   c. 2  g. 6
   d. 3  h. 7
3. 

\[ \begin{array}{ccccccc}
3 & 2 & 1 & 0 & 1 & 2 & 3 \\
\Delta & & & & x & & \\
\end{array} \]

a. 0  
\( \Delta \)  
\( x \)  
e. 4  

b. 1  
f. 5  
c. 2  
g. 6  
d. 3  
h. 7  

4. 

\[ \begin{array}{ccccccc}
3 & 2 & 1 & 0 & 1 & 2 & 3 \\
x & & & \Delta & & & \\
\end{array} \]

\( \Delta \)  
\( 7 \)  
a. 0  
e. 4  
b. 1  
f. 5  
c. 2  
g. 6  
d. 3  
h. 7  

5. 

\[ \begin{array}{ccccccc}
3 & 2 & 1 & 0 & 1 & 2 & 3 \\
x & & & \Delta & & & \\
\end{array} \]

\( \Delta \)  
\( 6 \)  
a. 0  
e. 4  
b. 1  
f. 5  
c. 2  
g. 6  
d. 3  
h. 7  

Task C1

Choose the number which balances the balance beam:

Example:

\[ \begin{array}{cccccc}
3 & 2 & 1 & 0 & 1 & 2 & 3 \\
\end{array} \]

\[ \begin{array}{c}
7 \\
\end{array} \]

\[ \begin{array}{c}
\triangle \\
\end{array} \]

\[ \begin{array}{c}
3 + x \\
\end{array} \]

\[ \begin{array}{cc}
M & M \\
M & M \\
M & M \\
M & M \\
M & M \\
\end{array} \]

The numbers in the circles below the beam are the weights.
The numbers to the left and right of the balance point (fulcrum, \( \triangle \)) are the distances:
The value \( x \) is the amount of weight needed to balance the beam. In the example, the amount needed is 4.

What size weight will make the beam balance? That weight goes at the question mark.

Solution:

\[ \begin{array}{cccccc}
3 & 2 & 1 & 0 & 1 & 2 & 3 \\
\end{array} \]

\[ \begin{array}{c}
7 \\
\end{array} \]

\[ \begin{array}{c}
\triangle \\
\end{array} \]

\[ \begin{array}{c}
3 + 4 \\
\end{array} \]

Circle the number that will make the beam balance:

1.

\[ \begin{array}{cccccc}
3 & 2 & 1 & 0 & 1 & 2 & 3 \\
\end{array} \]

\[ \begin{array}{c}
1 + ? \\
\end{array} \]

\[ \begin{array}{c}
\triangle \\
\end{array} \]

\[ \begin{array}{c}
3 \\
\end{array} \]

a. 0  e. 4  

b. 1  f. 5  
c. 2  g. 6  
d. 3  h. 7  

2.

\[ \begin{array}{cccccc}
3 & 2 & 1 & 0 & 1 & 2 & 3 \\
\end{array} \]

\[ \begin{array}{c}
2 + ? \\
\end{array} \]

\[ \begin{array}{c}
\triangle \\
\end{array} \]

\[ \begin{array}{c}
5 \\
\end{array} \]

a. 0  e. 4  

b. 1  f. 5  
c. 2  g. 6  
d. 3  h. 7  

3. 

\[ \begin{array}{ccccccc}
3 & 2 & 1 & 0 & 1 & 2 & 3 \\
\end{array} \]

\[ \Delta \]

3 +? 4 

\begin{itemize}
  \item a. 0
  \item b. 1
  \item c. 2
  \item d. 3
  \item e. 4
  \item f. 5
  \item g. 6
  \item h. 7
\end{itemize}

4. 

\[ \begin{array}{ccccccc}
3 & 2 & 1 & 0 & 1 & 2 & 3 \\
\end{array} \]

\[ \Delta \]

4 +? 2 

\begin{itemize}
  \item a. 0
  \item b. 1
  \item c. 2
  \item d. 3
  \item e. 4
  \item f. 5
  \item g. 6
  \item h. 7
\end{itemize}

5. 

\[ \begin{array}{ccccccc}
3 & 2 & 1 & 0 & 1 & 2 & 3 \\
\end{array} \]

\[ \Delta \]

6 +? 3 

\begin{itemize}
  \item a. 0
  \item b. 1
  \item c. 2
  \item d. 3
  \item e. 4
  \item f. 5
  \item g. 6
  \item h. 7
\end{itemize}
Task A1

What number makes the beam balance?

Example:

```
  3  2  1  0  1  2  3
  △   x
```

The numbers in the circles below the beam are the weights.
The numbers to the left and right of the balance point (fulcrum, △) are the distances:
The value x is the amount of weight needed to balance the beam. In the example, the amount needed is 3.

Solution:

```
  3  2  1  0  1  2  3
  △   x
```

Circle the correct answer:

1.

```
  3  2  1  0  1  2  3
  △   x
```

a. 0  e. 4
b. 1  f. 5
c. 2  g. 6
d. 3  h. 7

2.

```
  3  2  1  0  1  2  3
  △   x
```

a. 0  e. 4
b. 1  f. 5
c. 2  g. 6
d. 3  h. 7

3.

```
  4  3  2  1  0  1  2  3  4
  △   x
```

a. 0  e. 4
b. 1  f. 5
c. 2  g. 6
d. 3  h. 7
4. 

<table>
<thead>
<tr>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Δ</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. 0  

b. 1  
c. 2  
d. 3  
e. 4  
f. 5  
g. 6  
h. 7  

5. 

<table>
<thead>
<tr>
<th>5</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td></td>
<td></td>
<td></td>
<td>Δ</td>
<td></td>
<td></td>
<td>x</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

y= 4

a. 0  

b. 1  
c. 2  
d. 3  
e. 4  
f. 5  
g. 6  
h. 7  

**Task F1**

The diagrams below represent balance beams in a state of balance:

**Example 1**

```
5 4 3 2 1 0 1 2 3 4 5
```

```
3 6
```

M The numbers in the circles below the beam are the weights.
M The numbers to the left and right of the balance point (fulcrum, Δ) are the distances:
M The value x is the amount of weight needed to balance the beam. In the example, the amount needed is 4.

In a typical problem, you will be asked to solve for the unknown value, x:

**Example 3:**

```
5 4 3 2 1 0 1 2 3 4 5
```

```
4 1 x
```

The unknown value is 11:

**Solution:**

```
5 4 3 2 1 0 1 2 3 4 5
```

```
4 1 11
```

In problems 1-3, solve for the unknown weight, x:

1.

```
5 4 3 2 1 0 1 2 3 4 5
```

```
6
```

The unknown weight x is equal to

a. 3  e. 7
b. 4  f. 8
c. 5  g. 9
d. 6  h. 12

2.

```
5 4 3 2 1 0 1 2 3 4 5
```

```
6
```

The unknown weight x is equal to

a. 3  e. 7
The unknown weight $x$ is equal to
a. 2  e. 12
b. 3  f. 14
c. 5  g. 15
d. 10  h. 21

Beams 4 and 5 are not now in balance. You are given a specific amount of additional weight. You must determine the place where the additional weight is to be hung from the balance beam so that the balance beam balances. This place is described by its distance to the left or right of 0.

The additional weight is: 5

Place the weight:
a. 1 unit to the left of 0.  e. 1 unit to the right of 0.
b. 2 units to the left of 0.  f. 2 units to the right of 0.
c. 3 units to the left of 0.  g. 3 units to the right of 0.
d. 4 units to the left of 0.  h. 4 units to the right of 0.
The additional weight is 6. Place the weight

a. 1 unit to the left of 0.  

b. 2 units to the left of 0.  

c. 3 units to the left of 0.  

d. 4 units to the left of 0.  

e. 2 unit to the right of 0.  

f. 3 units to the right of 0.  

g. 4 units to the right of 0.  

h. 5 units to the right of 0.
Task F2

Once upon a time there was this strange school on another planet. There could even be negative numbers of students. The school used a very strange method for balancing the number of strong and weak students in a class.

- All the students were rated on a scale of -100 to 100 (negative one hundred to positive one hundred).
- The average rating of the students was always maintained at 0 in each class.
- When the class average was not 0, the balance between strong and weak students was upset. Then, the principal added new students so the class average was 0 again.

Sample Problem:

In one class, two students each received a rating of 4. How many students had to receive a rating of -8 in order to maintain the 0 average?

<table>
<thead>
<tr>
<th>Ratings</th>
<th>-8</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Students</td>
<td>?</td>
<td>2</td>
</tr>
</tbody>
</table>

Solution:

Only one student had to receive a rating of -8 in order to maintain the 0 average.

<table>
<thead>
<tr>
<th>Ratings</th>
<th>-8</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Students</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

Circle the correct answer:

1. In one class, two students each received a rating of 26. How many students had to receive a rating of -13 in order to maintain the 0 average?

<table>
<thead>
<tr>
<th>Ratings</th>
<th>-13</th>
<th>26</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Students</td>
<td>?</td>
<td>2</td>
</tr>
</tbody>
</table>

- a. -13  e. 4  
- b. -4   f. 13  
- c. -2   g. 26  
- d. 2    h. 28
2. In one class, two students each received a rating of -15. How many students had to receive a rating of 6 in order to maintain the 0 average?

<table>
<thead>
<tr>
<th>Ratings</th>
<th>-15</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Students</td>
<td>2</td>
<td>?</td>
</tr>
</tbody>
</table>

a. -5  
b. 3  
c. 5  
d. 7  
e. 8  
f. 12  
g. 13  
h. 15

3. In another class, the teacher gave 8 students a rating of 11 and 12 students a rating of -4. What rating would 8 more students have to receive in order for the average rating to remain at 0?

<table>
<thead>
<tr>
<th>Ratings</th>
<th>?</th>
<th>-4</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Students</td>
<td>8</td>
<td>12</td>
<td>8</td>
</tr>
</tbody>
</table>

a. -5  
b. 2  
c. 5  
d. 7  
e. 8  
f. 11  
g. 12  
h. 39

4. In yet another class, the teacher gave 3 students a rating of 6 and 4 students a rating of 12. How many students had to receive a rating of -6 in order for the average rating to be equal to 0?

<table>
<thead>
<tr>
<th>Ratings</th>
<th>-6</th>
<th>6</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Students</td>
<td>?</td>
<td>6</td>
<td>12</td>
</tr>
</tbody>
</table>

a. 1  
b. 3  
c. 5  
d. 6  
e. 7  
f. 11  
g. 11\frac{1}{4}  
h. 42

5. In yet another class, the teacher gave 4 students a rating of 7, 5 students a rating of 3, and 10 students a rating of -5. An unknown number of students joined the class, each with a rating of 1. How many students would have had to join the class in order for the average rating to be 0?

<table>
<thead>
<tr>
<th>Ratings</th>
<th>-5</th>
<th>1</th>
<th>3</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Students</td>
<td>10</td>
<td>?</td>
<td>5</td>
<td>4</td>
</tr>
</tbody>
</table>

a. -10  
b. -2  
c. 3  
d. 7  
e. 13  
f. 14  
g. 15  
h. 70
Task S1

You are still on Counter-Earth. Your mission now is to figure out how to make these beams balance. Here are two sample balance beams. The arithmetic operation that defines torque is the same for each. The mathematical relationship that governs the balancing of each system is also the same for each.

1. 

\[
\begin{array}{cccccccc}
7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\hline
x & \Delta & 10 & y \\
\end{array}
\]

The unknown weight \( x \) is equal to:  
\[ a. \quad 5 \]

The unknown weight \( y \) is equal to:  
\[ a. \quad 1 \]

2. 

\[
\begin{array}{cccccccc}
7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\
\hline
\Delta & x & y & 5 \\
\end{array}
\]

The unknown weight \( x \) is equal to:  
\[ a. \quad 5 \]

The unknown weight \( y \) is equal to:  
\[ a. \quad 1 \]

The diagrams below represent balance beams. The torque(s) on the left side of the balance beam balance the torque(s) on the right side, where torque is equal to weight times distance from the fulcrum (i.e., distance from 0). Solve for the unknown weights.

1. 

\[
\begin{array}{cccccccc}
5 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 5 \\
\hline
x+1 & \Delta & y \\
\end{array}
\]

\[
\begin{array}{cccccccc}
5 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 5 \\
\hline
x & \Delta & y & 1 \\
\end{array}
\]

The unknown weight \( x \) is equal to:  
\[ a. \quad -5 \quad e. \quad 8 \quad a. \quad -1 \quad e. \quad 6 \]

The unknown weight \( y \) is equal to:  
\[ b. \quad 3 \quad f. \quad 9 \quad b. \quad -2 \quad f. \quad 8 \]

\[ c. \quad 5 \quad g. \quad 12 \quad c. \quad 2 \quad g. \quad 9 \]

\[ d. \quad 7 \quad h. \quad 15 \quad d. \quad 4 \quad h. \quad 18 \]

2. 

\[
\begin{array}{cccccccc}
5 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 5 \\
\hline
x & \Delta & y & 1 \\
\end{array}
\]

\[
\begin{array}{cccccccc}
5 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 5 \\
\hline
x & \Delta & 7 & y \\
\end{array}
\]

The unknown weight \( x \) is equal to:  
\[ a. \quad -5 \quad e. \quad 8 \quad a. \quad -1 \quad e. \quad 6 \]

The unknown weight \( y \) is equal to:  
\[ b. \quad 3 \quad f. \quad 9 \quad b. \quad -2 \quad f. \quad 8 \]

\[ c. \quad 5 \quad g. \quad 12 \quad c. \quad 2 \quad g. \quad 9 \]

\[ d. \quad 7 \quad h. \quad 15 \quad d. \quad 4 \quad h. \quad 18 \]
Task S2 Fulcrum Task

The following balance beam is not balanced. Move the point of balance (Δ) so that it is balanced.

M You may not move the weights or change their value.

M HINT: The distances between the weights remain fixed.
These distances are shown on the beams below.

1.   5 4 3 2 1 0 1 2 3 4 5

Move the center of balance (Δ)

a. 1 units to the left.  
b. 2 units to the left.  
c. 3 units to the left.  
d. 4 units to the left.  
e. 1 unit to the right.  
f. 2 units to the right.  
g. 3 units to the right.  
h. 4 units to the right.

2.   7 6 5 4 3 2 1 0 1 2 3 4 5 6 7

Move the center of balance (Δ)

a. 1 units to the left.  
b. 2 units to the left.  
c. 3 units to the left.  
d. 4 units to the left.  
e. 1 unit to the right.  
f. 2 units to the right.  
g. 3 units to the right.  
h. 4 units to the right.
Task S3

1. In the same school on the other planet, one teacher taught only two classes: A morning class and an afternoon class. She rated each student on a scale of -100 to 100. The class average was 0 for both classes. These are her rosters. What are the two unknown ratings, \( x \) and \( y \)?

<table>
<thead>
<tr>
<th>Morning class</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ratings</strong></td>
<td>( x )</td>
<td>( y )</td>
</tr>
<tr>
<td><strong>Number of Students</strong></td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>( -3 )</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Afternoon class</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ratings</strong></td>
<td>( x )</td>
<td>( y )</td>
</tr>
<tr>
<td><strong>Number of Students</strong></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>( 1 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The unknown rating \( x \) is equal to:
- a. -5  e. 8  b. 3  f. 9  c. 5  g. 12  d. 7  h. 15

The unknown rating \( y \) is equal to:
- a. -9  e. 6  b. -1  f. 8  c. 2  g. 9  d. 4  h. 18
2. Another teacher taught two classes: one in the morning and one in the afternoon. The average rating for both classes was also 0. Solve for the two unknowns, x and y.

<table>
<thead>
<tr>
<th>Ratings</th>
<th>x</th>
<th>y</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Number of Students</strong></td>
<td>3</td>
<td>-5</td>
<td>-1</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ratings</th>
<th>x</th>
<th>y</th>
<th>-2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Number of Students</strong></td>
<td>4</td>
<td>-9</td>
<td>1</td>
</tr>
</tbody>
</table>

The unknown rating x is equal to:  
a. -5  
e. 8  
b. 3  
f. 9  
c. 5  
g. 12  
d. 7  
h. 15

The unknown rating y is equal to:  
a. -1  
e. 6  
b. -2  
f. 8  
c. 2  
g. 9  
d. 4  
h. 18
Task M1: Instructions

In this task, rate how similar pairs of systems are. These different systems consist of balance beams, student rating problems, and truck problems. Each system comes from the preceding parts of this questionnaire.

- Each system contains a Subsystem A and a Subsystem B.
- There are relationships between the subsystems. You need to examine them, looking at the things that the subsystems have in common with one another.

Remember, for all the student-ratings problems the following apply. Once upon a time there was this strange school on another planet. There could even be negative numbers of students. The school used very strange method for balancing the number of strong and weak students in a class.

- All the students were rated on a scale of -100 to 100 (negative one hundred to positive one hundred).
- The average rating of the students was always maintained at 0 in each class.
- When the class average was not 0, the balance between strong and weak students was upset. Then, the principal added new students so the class average was 0 again.

Please study all the systems carefully before answering.

- Solve for the unknown values in each system unless they are already solved.
- Describe the properties of the systems.
- Use the M1 answer sheet for your answers.
- Rate all the possible pairs of systems as to their similarity.
  - High ratings indicate similarity.
  - Low ratings indicate differentness.
- Explain your reasons for saying how similar.

On the following pages there is a sample problem with only three systems

- They happen to be balance beams but do not have to be.
- This example is to help you with the main problem (M1).
Sample Problem with Three Systems, Task M1

Sample System 1

<table>
<thead>
<tr>
<th>Subsystem A</th>
<th>Subsystem B</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 4 3 2 1 0 1 2 3 4 5</td>
<td>5 4 3 2 1 0 1 2 3 4 5</td>
</tr>
</tbody>
</table>

Sample System 2

<table>
<thead>
<tr>
<th>Subsystem A</th>
<th>Subsystem B</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 4 3 2 1 0 1 2 3 4 5</td>
<td>5 4 3 2 1 0 1 2 3 4 5</td>
</tr>
</tbody>
</table>

Sample System 3

<table>
<thead>
<tr>
<th>Subsystem A</th>
<th>Subsystem B</th>
</tr>
</thead>
<tbody>
<tr>
<td>5 4 3 2 1 0 1 2 3 4 5</td>
<td>5 4 3 2 1 0 1 2 3 4 5</td>
</tr>
</tbody>
</table>

On the next page, there is a sample answer sheet containing suggested ratings. These ratings tell you how similar or dissimilar the system pairs are.
Sample Answer Sheet

<table>
<thead>
<tr>
<th>Systems compared</th>
<th>Check if most similar</th>
<th>Similarity ratings</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0 1 2 3 4 5 6 7</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Dissimilar</td>
</tr>
<tr>
<td><strong>The suggested responses are in bold italics.</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>1</th>
<th>✓</th>
<th>0 1 2 3 4 5 6 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 &amp; 3</td>
<td></td>
<td>0 1 2 3 4 5 6 7</td>
</tr>
<tr>
<td>2 &amp; 3</td>
<td></td>
<td>0 1 2 3 4 5 6 7</td>
</tr>
</tbody>
</table>
Task M1

System 1

Subsystem A  In one class, x + 6 students each received a rating of 1. The other 5 students received a rating of x.

<table>
<thead>
<tr>
<th>Ratings</th>
<th>1</th>
<th>x</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Students</td>
<td>x + 6</td>
<td>5</td>
</tr>
</tbody>
</table>

Subsystem B  In one class, y + 6 students each received a rating of 500. The other 5 students received a rating of 500y.

<table>
<thead>
<tr>
<th>Ratings</th>
<th>500</th>
<th>500y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Students</td>
<td>y + 6</td>
<td>5</td>
</tr>
</tbody>
</table>

x = ?
y = ?

System 2

Subsystem A

<table>
<thead>
<tr>
<th>7</th>
<th>6</th>
<th>5</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>x Δ</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>y</td>
</tr>
</tbody>
</table>

Subsystem B  In one class, x students each received a rating of 6. The other x students received a rating of -5 -y.

<table>
<thead>
<tr>
<th>Ratings</th>
<th>6</th>
<th>-5 -y</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Students</td>
<td>x</td>
<td>5</td>
</tr>
</tbody>
</table>

x = ?
y = ?
System 3

Subsystem A

<table>
<thead>
<tr>
<th>6</th>
<th>5</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
</table>

x

6

y

Subsystem B

This truck is poorly designed to bear weight. Recently, the trucking company came up with a new system that allowed them to stack more freight toward the rear of the trailer and thereby increase the size of each shipment: A heavy weight was placed inside the trailer all the way to the front. A 500 lb weight placed up front balances 3000 lbs of freight loaded in back, provided that everything in the trailer is firmly secured.

x = ?
y = ?

System 4

Subsystem A

In one class, the strong students received a rating of y. The teacher thought the other x students should receive a rating of -8. How many were in this second group of students in order for the balance between strong and weak students to be maintained?

<table>
<thead>
<tr>
<th>Ratings</th>
<th>y</th>
<th>-8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Students</td>
<td>2</td>
<td>x</td>
</tr>
</tbody>
</table>

The same x and the y are in this balance beam.

Subsystem B

<table>
<thead>
<tr>
<th>6</th>
<th>5</th>
<th>4</th>
<th>3</th>
<th>2</th>
<th>1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
</table>

y

x = ?
y = ?
System 5

Subsystem A

\[ y \Delta 1 \]

\[ +x \]

Subsystem B

\[ 1 \Delta y \]

\[ +x \]

\[ y = 1 \]

\[ x = ? \]
## The Answer Sheet

### Answers for Task M: Compare the 5 system pairs to one another.

<table>
<thead>
<tr>
<th>Systems compared</th>
<th>Check if most similar</th>
<th>Similarity Rating:</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0 1 2 3 4 5 6 7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>0</strong></td>
<td></td>
<td>Dissimilar</td>
<td>Most similar</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 &amp; 2</td>
<td></td>
<td>0 1 2 3 4 5 6 7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 &amp; 3</td>
<td></td>
<td>0 1 2 3 4 5 6 7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 &amp; 4</td>
<td></td>
<td>0 1 2 3 4 5 6 7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 &amp; 5</td>
<td></td>
<td>0 1 2 3 4 5 6 7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 &amp; 3</td>
<td></td>
<td>0 1 2 3 4 5 6 7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 &amp; 4</td>
<td></td>
<td>0 1 2 3 4 5 6 7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 &amp; 5</td>
<td></td>
<td>0 1 2 3 4 5 6 7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 &amp; 4</td>
<td></td>
<td>0 1 2 3 4 5 6 7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 &amp; 5</td>
<td></td>
<td>0 1 2 3 4 5 6 7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 &amp; 5</td>
<td></td>
<td>0 1 2 3 4 5 6 7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### The Answer Sheet 2 Optional

#### Answers for Task M: Compare the 5 system pairs to one another.

<table>
<thead>
<tr>
<th>Systems compared</th>
<th>Check which pair of pairs is most similar</th>
<th>Pairs of Systems 1 is _____________ than Pair 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Dissimilar    Same Similarity    More Similar</td>
</tr>
<tr>
<td>1 &amp; 2 or 1 &amp; 3</td>
<td></td>
<td>1              2              3</td>
</tr>
<tr>
<td>1 &amp; 2 or 1 &amp; 4</td>
<td></td>
<td>1              2              3</td>
</tr>
<tr>
<td>1 &amp; 2 or 1 &amp; 5</td>
<td></td>
<td>1              2              3</td>
</tr>
<tr>
<td>2 &amp; 1 or 2 &amp; 3</td>
<td></td>
<td>1              2              3</td>
</tr>
<tr>
<td>2 &amp; 1 or 2 &amp; 4</td>
<td></td>
<td>1              2              3</td>
</tr>
<tr>
<td>2 &amp; 1 or 2 &amp; 5</td>
<td></td>
<td>1              2              3</td>
</tr>
<tr>
<td>1 &amp; 3 or 1 &amp; 4</td>
<td></td>
<td>1              2              3</td>
</tr>
<tr>
<td>1 &amp; 3 or 1 &amp; 5</td>
<td></td>
<td>1              2              3</td>
</tr>
<tr>
<td>3 &amp; 1 or 3 &amp; 2</td>
<td></td>
<td>1              2              3</td>
</tr>
<tr>
<td>3 &amp; 1 or 3 &amp; 4</td>
<td></td>
<td>1              2              3</td>
</tr>
<tr>
<td>3 &amp; 1 or 3 &amp; 5</td>
<td></td>
<td>1              2              3</td>
</tr>
<tr>
<td>1 &amp; 4 or 1 &amp; 5</td>
<td></td>
<td>1              2              3</td>
</tr>
<tr>
<td>4 &amp; 1 or 4 &amp; 2</td>
<td></td>
<td>1              2              3</td>
</tr>
<tr>
<td>4 &amp; 1 or 4 &amp; 3</td>
<td></td>
<td>1              2              3</td>
</tr>
<tr>
<td>4 &amp; 1 or 4 &amp; 5</td>
<td></td>
<td>1              2              3</td>
</tr>
<tr>
<td>5 &amp; 1 or 5 &amp; 2</td>
<td></td>
<td>1              2              3</td>
</tr>
<tr>
<td>5 &amp; 1 or 5 &amp; 3</td>
<td></td>
<td>1              2              3</td>
</tr>
<tr>
<td>5 &amp; 1 or 5 &amp; 4</td>
<td></td>
<td>1              2              3</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>----------------</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>2 &amp; 3 or 2 &amp; 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 &amp; 3 or 2 &amp; 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 &amp; 2 or 3 &amp; 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 &amp; 2 or 3 &amp; 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 &amp; 4 or 2 &amp; 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 &amp; 2 or 4 &amp; 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 &amp; 2 or 4 &amp; 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 &amp; 2 or 5 &amp; 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 &amp; 2 or 5 &amp; 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 &amp; 4 or 3 &amp; 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 &amp; 3 or 4 &amp; 5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

State your reasons below.

_______________________________________________________________________________________
_______________________________________________________________________________________
_______________________________________________________________________________________
_______________________________________________________________________________________
_______________________________________________________________________________________
_______________________________________________________________________________________
_______________________________________________________________________________________
_______________________________________________________________________________________
Motivation For Doing Problems

Subject number ___________________________ Date ____________________________

Last 4 Digits of Your Social Security Number Month Day Year

Name of Administrator of problem __________________________________________________________________________ Please print.

1. Please state your reasons why you thought you should do these problems.

2. Please state your reasons why you thought you should do these problems well.

3. How much total time did you spend on doing this problems?
   First time:
   4. At what time did you start working on these problems? _____
   5. At what time did you stop? ______
   6. What problem had you started when you stopped?
   7. Why did you stop?

   Second time:
   8. At what time did you start working on these problems _____
   9. At what time did you start stop ______
   10. Why did you stop?

11. To what degree did doing well matter to you ____________________________
   Not at all 0 1 2 3 4 5 6 7 Totally
   12. Why?

13. I am the ___ of the researcher?
   __Student of ___Student in the school ___Student of a Colleague
   ___Not related ___Brother ___Sister ___Mother ___Father ___Cousin ___Uncle ___Aunt
   ___Nephew ___Niece ___Fiance ___Boy Friend ___Girl Friend ___Husband ___Wife ___Ex-spouse

14. If so, did your relationship to the researcher motivate you to try harder?
15. Was there any reward given to you after you finished the problems?
16. If so, what was it?

17. My grade point average in high school was ___D__, ___C- __, ___C__ , ___C+__ ___B-__, ___B__, ___B+__, ___A- __, ___A__
18. My grade point average in my 1st college was ___D__, ___C- __, ___C__ , ___C+__ ___B-__, ___B__, ___B+__, ___A- __, ___A__
19. My grade point average in my 2nd college was ___D__, ___C- __, ___C__ , ___C+__ ___B-__, ___B__, ___B+__, ___A- __, ___A__

My grade was in:

20. Algebra 1 Did not take___, F___, D___, C-___, C___, C+___ B-___, B___, B+___, A-___, A___
21. Algebra 2 Did not take___, F___, D___, C-___, C___, C+___ B-___, B___, B+___, A-___, A___
22. Geometry 1 Did not take___, F___, D___, C-___, C___, C+___ B-___, B___, B+___, A-___, A___
23. Geometry 2 Did not take___, F___, D___, C-___, C___, C+___ B-___, B___, B+___, A-___, A___
24. PreCalculus Did not take___, F___, D___, C-___, C___, C+___ B-___, B___, B+___, A-___, A___
25. Trigonometry Did not take___, F___, D___, C-___, C___, C+___ B-___, B___, B+___, A-___, A___
26. College Algebra Did not take___, F___, D___, C-___, C___, C+___ B-___, B___, B+___, A-___, A___
27. Calculus 1 Did not take___, F___, D___, C-___, C___, C+___ B-___, B___, B+___, A-___, A___
28. Calculus 2 Did not take___, F___, D___, C-___, C___, C+___ B-___, B___, B+___, A-___, A___
29. Calculus 3 Did not take___, F___, D___, C-___, C___, C+___ B-___, B___, B+___, A-___, A___
30. Calculus 4 Did not take___, F___, D___, C-___, C___, C+___ B-___, B___, B+___, A-___, A___
31. Other Did not take___, F___, D___, C-___, C___, C+___ B-___, B___, B+___, A-___, A___
Please rate these possible reasons for doing the problems

<table>
<thead>
<tr>
<th></th>
<th>Strongly Disagree</th>
<th>Strongly Agree</th>
</tr>
</thead>
<tbody>
<tr>
<td>32. I was wide awake.</td>
<td>0 1 2 3 4 5 6 7</td>
<td></td>
</tr>
<tr>
<td>33. Good people do these problems.</td>
<td>0 1 2 3 4 5 6 7</td>
<td></td>
</tr>
<tr>
<td>34. Inner city kids will benefit from our doing these problems.</td>
<td>0 1 2 3 4 5 6 7</td>
<td></td>
</tr>
<tr>
<td>35. Other present students will benefit from our doing these problems.</td>
<td>0 1 2 3 4 5 6 7</td>
<td></td>
</tr>
<tr>
<td>36. Future students will benefit from our doing these problems.</td>
<td>0 1 2 3 4 5 6 7</td>
<td></td>
</tr>
<tr>
<td>37. I benefit from the results of research. Therefore, I should participate in it.</td>
<td>0 1 2 3 4 5 6 7</td>
<td></td>
</tr>
<tr>
<td>38. Doing this project will help the school.</td>
<td>0 1 2 3 4 5 6 7</td>
<td></td>
</tr>
<tr>
<td>39. Doing this project will help raise my grade or my friend's, relative's or roommate's grade.</td>
<td>0 1 2 3 4 5 6 7</td>
<td></td>
</tr>
<tr>
<td>40. If I do not do this, either the dean, my advisor or professor or my friend, relative or mate, etc will hate me.</td>
<td>0 1 2 3 4 5 6 7</td>
<td></td>
</tr>
<tr>
<td>41. This is a stupid study, but I have to do it because it is the rule.</td>
<td>0 1 2 3 4 5 6 7</td>
<td></td>
</tr>
<tr>
<td>42. If I do these problems, either the dean, my advisor or professor will think that I am a good student or that my friend, relative or mate is a good student.</td>
<td>0 1 2 3 4 5 6 7</td>
<td></td>
</tr>
<tr>
<td>43. Mainly, I do these problems so the school will not keep me from graduating.</td>
<td>0 1 2 3 4 5 6 7</td>
<td></td>
</tr>
<tr>
<td>44. As a student, I am interested in how adults develop.</td>
<td>0 1 2 3 4 5 6 7</td>
<td></td>
</tr>
<tr>
<td>45. This will give me some idea of how subjects experience being tested, so that when I become a professional, I will be able to do a more professional job.</td>
<td>0 1 2 3 4 5 6 7</td>
<td></td>
</tr>
<tr>
<td>46. I care about my school and want to help it.</td>
<td>0 1 2 3 4 5 6 7</td>
<td></td>
</tr>
<tr>
<td>47. I care about my associates school and want to help them.</td>
<td>0 1 2 3 4 5 6 7</td>
<td></td>
</tr>
<tr>
<td>48. Give us you inputs on the problems.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Many thanks to you for doing these problems. Your time was greatly appreciated.